The structure **ABCDE** shown in the figure 1, where A and B are roller supports, D a pin support and C a hinge joint, is subjected to the system of forces shown and is designed with the cross section sketch and properties of figure 2:

The material has the next properties:

- Maximum tensile admissible strength of the cross section material is **150 MPa**
- Maximum compressive admissible strength of the cross section material is **250 MPa**

Additionally, the value of the reaction forces at the supports is known and equal to:

H_a = 3.5 kN (right); V_b = 3.0 kN (upwards); H_d = 0.5 kN (left); V_d = 4.5 kN (upwards).

With all this information, for the beam and its corresponding cross section, **determine:**

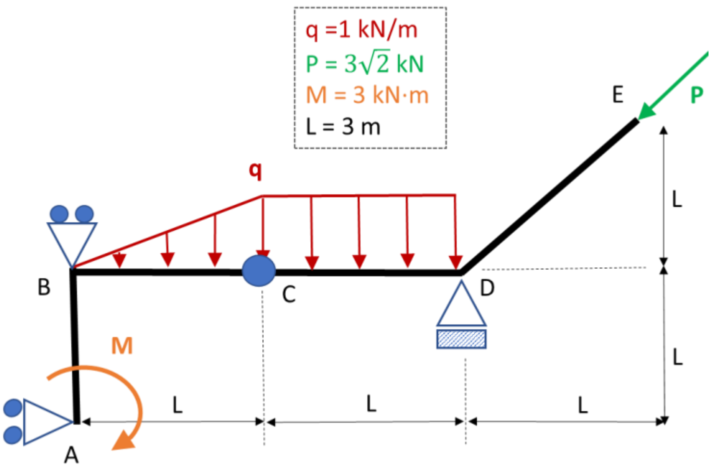


Figure 1

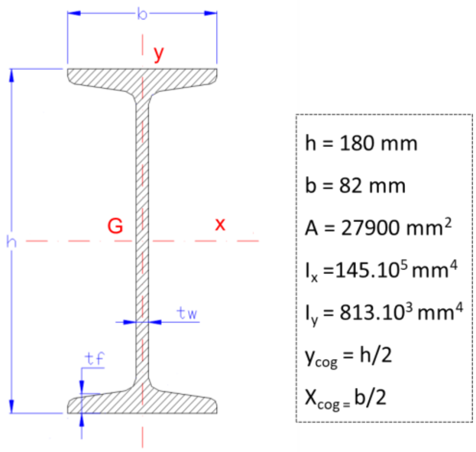


Figure 2

a) Axial force diagram at stretch BC and CD, including main/extreme values. Provide the analytical expressions.

b) Shear force diagram at stretch BC and CD, including main/extreme values. Provide the analytical expressions.

c) Bending moment diagram at stretch BC and CD, including main/extreme values. Provide the analytical expressions.

In relation with the stresses of the cross section of figure 2, **determine**:

d) Distribution of stress at section B with the cross section shown in figure 2 considering the normal force and the bending moment.

e) Safety coefficient of the structure in section B of the cross section for tensile and compressive state. Explain which type of failure would arise first (tensile or compressive).

Consider now the cross section shown in figure 2 rotated 90 degrees with respect the axis passing through its center of mass. For this new scenario, **determine:**

f) Distribution of stress at section B considering the normal force and the bending moment.

g) Safety coefficient of the structure in section B of the cross section for tensile and compressive state. Explain which type of failure would arise first (tensile or compressive state).

Finally considering both cross sections (figure 2 and figure 2 rotated), **determine:**

h) Which of the cross sections of the exercise is more suitable for the design of the structure with the loads of the statement. Do not base your answer exclusively in the safety coefficient.

SOLUTION:

Given the data of the statement about the value of the reaction forces:

H_a = 3.5 kN (right)

V_b = 3.0 kN (upwards)

H_d = 0.5 kN (left)

V_d = 4.5 kN (upwards)

[1] Force laws

The structure will be cut at the different stretches in order to obtain the internal forces:

$$\{(0 \leq x \leq 3) \cap (y = 3)\}$$

$$N = \mathbf{3,5\ kN\ (C)}$$

$$q(x) = \frac{x}{3}$$

$$V_T(x) = \int_0^x \frac{\xi}{3} d\xi = \frac{x^2}{6}$$

$$\mathbf{V(x) = 3 - \frac{x^2}{6}}$$

$$M_T(x) = \int_0^x \frac{\xi^2}{6} d\xi = \frac{x^3}{18}$$

$$\mathbf{M(x) = -7,5 + 3x - \frac{x^3}{18}}$$

$$M_{max} \to \frac{dM(x)}{dx} = V(x) = 0 \to x = \sqrt{18} \notin (0 \leq x \leq 3)$$

$$\{(3 \leq x \leq 6) \cap (y = 3)\}$$

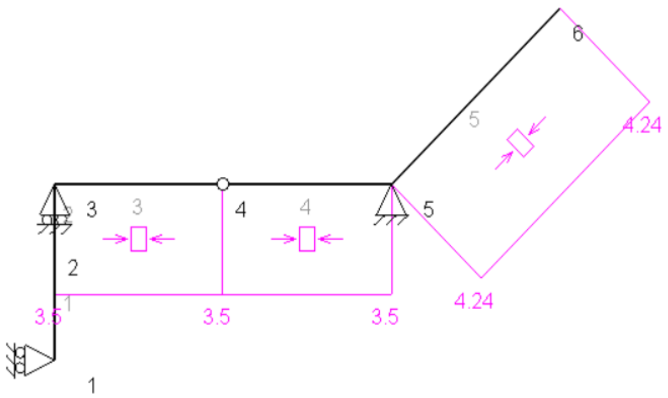
$$\mathbf{N = 3\ kN\ (C)}$$

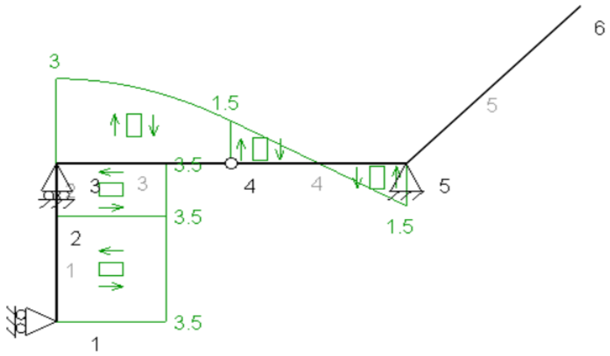
$$\mathbf{V(x) = 3 - \frac{3}{2} - (x-3) = 4,5 - x}$$

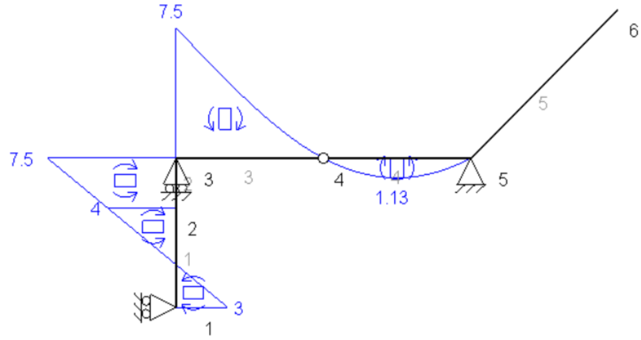
$$\mathbf{M(x) = -7,5 + 3x - \frac{3}{2}(x-2) - \frac{(x-3)^2}{2}}$$

$$M_{max} \to \frac{dM(x)}{dx} = V(x) = 0 \to x = 4,5\ \text{m}$$

$$M_{max} = 1,13\ \text{kN.m}$$







[2] Stress distribution cross section figure 2

$$\sigma_c = \frac{N_c}{A} + \frac{M_x . y}{I_x}$$

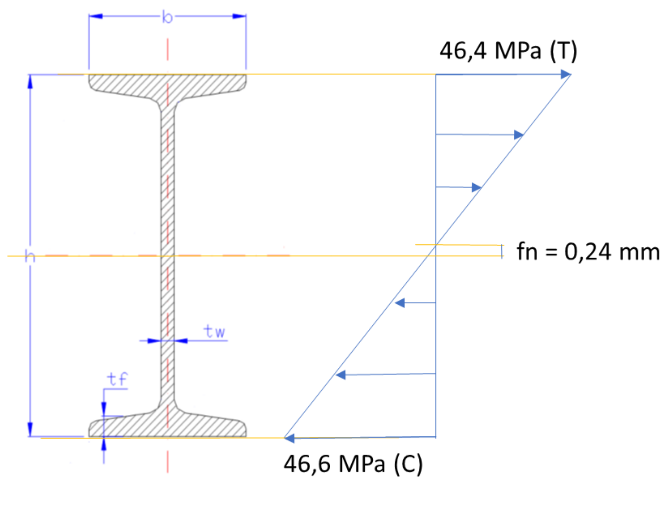
$$\sigma_c = \frac{3500\ N}{27900\ mm^2} \pm \frac{7,5.10^3 . N.m.y(mm)\ 10^{-3}m}{14,5\ .\ 10^{-6}m^4}$$

$$\sigma_c = 0,12 \pm 0,51.y(mm)$$

$$\sigma_c(y = 90) = 0,12 - 0,51.90(mm) = 46,4\ \text{MPa (T)}$$

$$\sigma_c(y = -90) = 0,12 + 0,51.90(mm) = 46,6\ \text{MPa (C)}$$

$$y_{fn} = \frac{0,12}{0,51} = 0,24\ \text{mm (above the cog)}$$



$$C_S^t = \frac{\sigma_{max\,tensile}}{\sigma_{service\,tensile}} = \frac{150\ \text{MPa}}{45{,}8\ \text{MPa}} = 3{,}27$$

$$C_S^c = \frac{\sigma_{max\,compressive}}{\sigma_{service\,compressive}} = \frac{250\ \text{MPa}}{46\ \text{MPa}} = 5{,}43$$

$C_S^c > C_S^t \rightarrow$ The structure will fail first under the tensile state of load, since the safety factor is lower.

[3] <u>Stress distribution cross section figure 2 rotated</u>

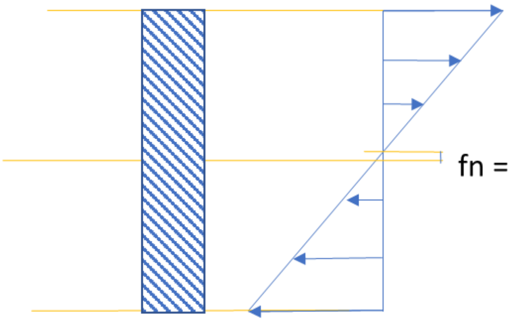
$$\sigma_c = \frac{3500\ \text{N}}{27900\ \text{mm}^2} \pm \frac{7{,}5.10^3.\text{N.m.y(mm)}\ 10^{-3}\text{m}}{0{,}813\ .\ 10^{-6}\text{m}^4}$$

$$\sigma_c = 0{,}12 \pm 9{,}22.\text{y(mm)}$$

$$\sigma_c(y = 41) = 0{,}12 - 9{,}22.41(\text{mm}) = 378{,}1\ \text{MPa (T)}$$

$$\sigma_c(y = -41) = 0{,}12 + 9{,}22.41(\text{mm}) = 378{,}3\ \text{MPa (C)}$$

$$y_{fn} = \frac{0{,}12}{9{,}22} = 0{,}013\ \text{mm (above the cog)}$$



The configuration won't withstand any of the tensional states. It makes no sense to talk about safety coefficient when the maximum value has been surpassed. However, their value is given below in order to ease the correction.

$$C_S^t = \frac{\sigma_{max}}{\sigma_{service\,tensile}} = \frac{150\ \text{MPa}}{377{,}9} = 0{,}396$$

$$C_S^c = \frac{\sigma_{max}}{\sigma_{service\,compressive}} = \frac{250}{378{,}1} = 0{,}661$$

$C_S^c > C_S^t \rightarrow$ The structure will fail first under the tensile state of load, since the safety factor is lower.

[4] <u>Best configuration</u>

<u>The first configuration (calculated at point [2]) is more efficient</u> than the one rotated because its mass is far away from the COG and therefore it will have a higher value of the inertia which also makes stress lower (note the contribution of the bending is much more important than the one corresponding to the normal force).

Rotated cross section is not adequate when dealing with normal stresses because all material is close to the axis, not even contributing to the value of the inertia, and therefore allowing higher values of the stress (in the exercise even surpassing the maximum of the material).